

NEUTRINOLESS DOUBLE β -DECAY: THE PROBLEM OF NUCLEAR MATRIX ELEMENTS ¹

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Abstract

The calculations of nuclear matrix elements of $0\nu\beta\beta$ -decay is a challenge for nuclear physics. We are discussing here a model independent method, which could allow to test the calculations. The method is based on the factorization property of the nuclear matrix elements and requires observation of neutrinoless double β -decay of several nuclei.

1 Introduction

The discovery of neutrino oscillations in the atmospheric Super Kamiokande [1], solar SNO [2], reactor KamLAND [3], accelerator K2K [4] and other neutrino oscillation experiments [5, 6, 7, 8] is one of the major recent discoveries in the elementary particle physics. All neutrino oscillation data with the exception of the data of the short baseline LSND experiment [9] are well described by the three-neutrino mixing ²

$$\nu_{lL} = \sum_{i=1}^3 U_{li} \nu_{iL} . \quad (1)$$

Here U is PMNS [11, 12] mixing matrix, ν_i is the field of neutrino with mass m_i and ν_{lL} ($l = e, \mu, \tau$) is neutrino field which enter into the standard charged and neutral currents (so called flavor neutrino field).

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²The LSND result will be checked in the near future by the running MiniBooNE experiment [10].

From the analysis of the SK atmospheric neutrino data for oscillation parameters $|\Delta m_{32}^2|$ and $\sin^2 2\theta_{23}$ the following 90% CL ranges were found [1]³

$$1.9 \leq |\Delta m_{32}^2| \leq 3 \cdot 10^{-3} \text{ eV}^2; \quad \sin^2 2\theta_{23} < 0.9 \quad (2)$$

From analysis of the data of the solar and the reactor KamLAND experiments for oscillation parameters Δm_{21}^2 and $\tan^2 \theta_{12}$ it was found [2]

$$\Delta m_{21}^2 = (8_{-0.4}^{+0.6}) \cdot 10^{-5} \text{ eV}^2; \quad \tan^2 \theta_{12} = 0.45_{-0.07}^{+0.09}. \quad (3)$$

From the exclusion plot obtained from the analysis of the data of the reactor CHOOZ experiment [13] for the parameter $\sin^2 \theta_{13}$ the following upper bound can be inferred

$$\sin^2 \theta_{13} < 5 \cdot 10^{-2} \quad (90\% \text{ CL}). \quad (4)$$

An information about the absolute values of neutrino masses can be obtained from experiments on the measurement of the high-energy part of the β -spectrum of tritium and from cosmological data. From the data of the Troitsk [14] and Mainz [15] tritium experiments it was found

$$m_0 < 2.2 \text{ eV}, \quad (5)$$

where m_0 is the mass of the lightest neutrino. From analysis of the cosmological data for the sum of the neutrino masses the upper bound in the range

$$\sum_i m_i < (1 - 2) \text{ eV}. \quad (6)$$

was obtained (see [16]).

The nature of the massive neutrinos (Dirac or Majorana?) is at present unknown. The progress in the understanding of the origin of the neutrino masses and mixing strongly depends on the answer to this fundamental question. In particular, the proof that massive neutrinos ν_i are Majorana particles would provide a strong argument in favor of the famous see-saw mechanism which, apparently, is the most natural mechanism of the generation of small neutrino masses.

³The neutrino mass-squared difference is determined as follows: $\Delta m_{ik}^2 = m_i^2 - m_k^2$. In the case of normal mass spectrum neutrino masses are labeled in such a way that $m_1 < m_2 < m_3$. In this case $\Delta m_{32}^2 > 0$. In the case of inverted neutrino mass spectrum neutrino masses are labeled as follows $m_3 < m_1 < m_2$. In this case $\Delta m_{32}^2 < 0$.

2 Neutrinoless double β -decay

To reveal the nature of the massive neutrinos it is necessary to study processes in which the total lepton number $L = L_e + L_\mu + L_\tau$ is not conserved. The search for neutrinoless double β -decay ($0\nu\beta\beta$ -decay) of some even-even nuclei

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- \quad (7)$$

is the most sensitive method of the investigation of the nature of the massive neutrinos ν_i .

If $\nu_i(x)$ satisfies the Majorana condition

$$\nu_i(x) = \nu_i^c(x) = C \bar{\nu}_i^T(x), \quad (8)$$

(C is the matrix of the charge conjugation) the process (7) is allowed. In this case neutrinoless double β -decay is a process of the second order in the Fermi constant G_F with virtual neutrinos ν_i . The half-life of the process is given by the following general expression (see reviews [17])

$$\frac{1}{T_{1/2}(A, Z)} = |m_{ee}|^2 |M(A, Z)|^2 G(E_0, Z). \quad (9)$$

Here

$$m_{ee} = \sum_i U_{ei}^2 m_i \quad (10)$$

is the effective Majorana mass, $G(E_0, Z)$ is known phase-space factor (E_0 is the energy release) and $M(A, Z)$ is nuclear matrix element (NME). For light neutrino masses NME do not depend on m_i .

The results of many experiments on the search for $0\nu\beta\beta$ -decay are available at present (see [18]). The most stringent lower bounds on the half-life of the $0\nu\beta\beta$ -decay were obtained in the Heidelberg-Moscow ^{76}Ge experiment [19] and in the recent CUORICINO ^{130}Te cryogenic experiment [20]:

$$\begin{aligned} T_{1/2}(^{76}\text{Ge}) &\geq 1.9 \cdot 10^{25} \text{ y} \quad (\text{Heidelberg} - \text{Moscow}) \\ T_{1/2}(^{130}\text{Te}) &\geq 1.8 \cdot 10^{24} \text{ y} \quad (\text{CUORICINO}). \end{aligned} \quad (11)$$

Taking into account different calculations of nuclear matrix elements, for the effective Majorana mass $|m_{ee}|$ from these results the following upper bounds can be inferred

$$\begin{aligned} |m_{ee}| &\leq (0.3 - 1.2) \text{ eV} \quad (\text{Heidelberg} - \text{Moscow}) \\ |m_{ee}| &\leq (0.2 - 1.1) \text{ eV} \quad (\text{CUORICINO}) \end{aligned} \quad (12)$$

Many new experiments on the search for the neutrinoless double β -decay of different nuclei are in preparation at present (see [21]). In these future experiments two order of the magnitude improvement in the sensitivity to $|m_{ee}|$ is expected :

$$|m_{ee}| \simeq \text{a few} \cdot 10^{-2}. \quad (13)$$

3 Effective Majorana mass

The effective Majorana mass m_{ee} is determined by neutrino masses m_i and elements U_{ei} . In the standard parametrization we have

$$U_{e1} = \cos \theta_{13} \cos \theta_{12} e^{i\alpha_1}; \quad U_{e2} = \cos \theta_{13} \sin \theta_{12} e^{i\alpha_2}; \quad U_{e3} = \sin \theta_{13} e^{i\alpha_3}, \quad (14)$$

where α_i are Majorana CP phases.

From neutrino oscillation data we know the values of neutrino mass-squared differences $|\Delta m_{32}^2|$ and Δm_{21}^2 and the angle θ_{23} (see (2) and (3)). We know also that the angle θ_{13} is small (see (4)). We do not know the character of neutrino mass spectrum (normal or inverted), the mass of the lightest neutrino m_0 , which determine the absolute values of neutrino masses, and Majorana phases.

Let us consider three standard neutrino mass spectra (see [22])

1. Hierarchy of neutrino masses

$$m_1 \ll m_2 \ll m_3. \quad (15)$$

In this case we have

$$\begin{aligned} |m_{ee}| &\simeq \left| \sin^2 \theta_{12} \sqrt{\Delta m_{21}^2} + \sin^2 \theta_{13} \sqrt{\Delta m_{32}^2} e^{2i\alpha_{32}} \right| \\ &\leq \left(\sin^2 \theta_{12} \sqrt{\Delta m_{21}^2} + \sin^2 \theta_{13} \sqrt{\Delta m_{32}^2} \right), \end{aligned} \quad (16)$$

where $\alpha_{32} = \alpha_3 - \alpha_2$. Using the values (2) and (3) of neutrino oscillation parameters from (16) for effective Majorana mass we find the upper bound

$$|m_{ee}| \leq 6.4 \cdot 10^{-3} \quad (17)$$

Thus, in the case of the neutrino mass hierarchy the predicted upper bound of $|m_{ee}|$ is significantly smaller than the sensitivity of the most ambitious future experiments.

2. Inverted hierarchy of neutrino masses

$$m_3 \ll m_1 < m_2. \quad (18)$$

In this case we have

$$|m_{ee}| \simeq \sqrt{|\Delta m_{31}^2|} (1 - \sin^2 2\theta_{12} \sin^2 \alpha_{21})^{\frac{1}{2}} \quad (19)$$

From this relation we find

$$\sqrt{|\Delta m_{31}^2|} \cos 2\theta_{12} \leq |m_{ee}| \leq \sqrt{|\Delta m_{31}^2|} \quad (20)$$

Let us notice that the bounds in this inequality correspond to the case of the CP conservation in the lepton sector: the upper (lower) bound corresponds to the case of equal (opposite) CP-parities of ν_2 and ν_1 .

Thus, in the case of the inverted neutrino mass hierarchy from neutrino oscillation data we can predict the upper and lower bounds of the possible values of the effective Majorana mass. From (2), (3) and (20) we obtain the range

$$1.0 \cdot 10^{-2} \leq |m_{ee}| \leq 5.5 \cdot 10^{-2} \text{ eV} \quad (21)$$

The values of $|m_{ee}|$ in this range apparently will be reached in future $0\nu\beta\beta$ - experiments. From (19) for the parameter $\sin^2 \alpha_{21}$, which characterizes CP violation in the case of the Majorana neutrino mixing, we have

$$\sin^2 \alpha_{21} = \frac{1}{\sin^2 2\theta_{12}} \left(1 - \frac{|m_{ee}|^2}{|\Delta m_{31}^2|} \right) \quad (22)$$

Hence, the observation of the neutrinoless double β -decay with $|m_{ee}|$ in the range (21) could allow to obtain an information about Majorana CP phase difference.

3. Quasi-degenerate neutrino mass spectrum

$$m_1 < m_2 < m_3 \text{ } (m_3 < m_1 < m_2); \text{ } m_0 \gg \sqrt{|\Delta m_{32}^2|} \quad (23)$$

Neglecting small contribution of $\sin^2 \theta_{13}$ for the effective Majorana mass we have in this case

$$|m_{ee}| \simeq m_0 (1 - \sin^2 2\theta_{12} \sin^2 \alpha_{21})^{\frac{1}{2}} \quad (24)$$

In the future tritium experiment KATRIN [23] the sensitivity $m_0 \simeq 0.2$ eV is planned to be reached. If the mass of the lightest neutrino m_0 will be measured in this experiment for the effective Majorana mass from (3) and (24) we obtain the range

$$0.23 m_0 \leq |m_{ee}| \leq m_0. \quad (25)$$

On the other side if neutrinoless double β -decay will be observed in future experiments with the value of $|m_{ee}|$ which is larger than the upper bound (21) for the mass of the lightest neutrino we will have the bound

$$|m_{ee}| \leq m_0 \leq 4.4 |m_{ee}|. \quad (26)$$

4 Nuclear matrix elements

The observation of neutrinoless double β -decay would be a direct proof that ν_i are Majorana particles. As we have seen, the determination of the effective Majorana mass would allow to obtain an important information about character of the neutrino mass spectrum, mass of the lightest neutrino and, possibly, Majorana CP phase difference.

However, from the measurement of the half-life of $0\nu\beta\beta$ -decay only the *product* of the effective Majorana mass and nuclear matrix element can be determined. The calculation of NME is a complicated nuclear problem (see [17]). Two models are commonly used: Nuclear Shell Model (NSM) and Quasiparticle Random Phase Approximation (QRPA) with numerous modifications. These two approaches are based on different physical assumptions. As a result different calculations of the same NME differ by factor 2-3 or even more.

We will discuss now a possible method of a model independent test of NME calculations [24]. We will use only the general factorization property of matrix elements of the $0\nu\beta\beta$ decay. Namely the fact that the matrix element of the process is a product of the effective Majorana neutrino mass, which is determined by neutrino masses and mixing, and nuclear matrix element,

which (for light neutrinos) does not depend on neutrino masses. From (9) we find the following relations between half-lives of the $0\nu\beta\beta$ -decay of different nuclei:

$$\begin{aligned} T_{1/2}(A_1, Z_1) &= X(A_1, Z_1; A_2, Z_2) T_{1/2}(A_2, Z_2) = \\ &X(A_1, Z_1; A_3, Z_3) T_{1/2}(A_3, Z_3) = \dots \end{aligned} \quad (27)$$

Here

$$X(A_i, Z_i; A_k, Z_k) = \frac{|M(A_k, Z_k)|^2 G(E_0, Z_k)}{|M(A_i, Z_i)|^2 G(E_0, Z_i)} \quad (28)$$

The coefficients in relations (27) have to be calculated. If $0\nu\beta\beta$ -decay of *different nuclei* will be observed in future experiments and relations (27) with coefficients $X(A_i, Z_i; A_k, Z_k)$ calculated in some model M are satisfied than the model M is compatible with data (it is obvious that if relations (27) are not satisfied the corresponding model must be rejected). This does not mean, however, that the model M allows us to obtain the correct value of $|m_{ee}|$ from experimental data. In fact if nuclear matrix elements, calculated in the framework of different models M_a and M_b are proportional

$$|M_{M_a}(A, Z)| = \beta |M_{M_b}(A, Z)|, \quad (29)$$

(β is a coefficient which does not depend on (A, Z)) and relations (27) are satisfied for the model M_a than obviously they are satisfied also for the model M_b . The values of the effective Majorana mass, which can be determined from experimental data in the framework of these two models, are connected by the relation

$$|m_{ee}|_{M_a} = \frac{1}{\beta} |m_{ee}|_{M_b} \quad (30)$$

and could be quite different.

For the purpose of illustration we will calculate the coefficients in Eq. (27) in three different recent models of the calculation of nuclear matrix elements of $0\nu\beta\beta$ -decay: RFSV [25], CS [26] and NSM [27]. In the paper [25], based on QRPA and renormalized QRPA approaches, the values of the parameter of the particle-particle interaction g_{pp} , determined from the measured half-lives of the $2\nu\beta\beta$ -decay of corresponding nuclei, were used. In the QRPA calculation [26] the values of the parameters were determined from the data on the β -decay of the nuclei of the interest for double β -decay transitions. In the paper [27] the results of the latest NSM calculations were given. We will

consider four different nucleus: ^{76}Ge , ^{100}Mo , ^{130}Te and ^{136}Xe . In the Table I we have presented the values of coefficients $X(A_i, Z_i; A_k, Z_k)$ in the case if $0\nu\beta\beta$ -decay of ^{76}Ge and one of the other nucleus is observed.

Table I

The values of the coefficient $X(A_i, Z_i; A_k, Z_k)$ obtained with NME calculated in [25] (RFSV), in [26] (CS) and in [27] (NSM).

	<i>RFSV</i>	<i>CS</i>	<i>NSM</i>
$X(^{130}\text{Te}; ^{76}\text{Ge})$	0.38	0.13	0.24
$X(^{136}\text{Xe}; ^{76}\text{Ge})$	0.80	0.07	0.56
$X(^{100}\text{Mo}; ^{76}\text{Ge})$	0.59	0.17	—

If it would occur that the relation (27) with the coefficient, calculated in one of the model considered, is satisfied, in this case, as we can see from the Table I, other models apparently can be excluded (if accuracy of experimental data are better than $\simeq 30\%$)

Let us stress that this conclusion depends on nuclei for which neutrinoless double β -decay is observed. For example, if $0\nu\beta\beta$ -decay of ^{130}Te and ^{100}Mo will be observed, in this case we have

$$X(^{100}\text{Mo}; ^{76}\text{Ge}) = 1.5 \text{ (RFSV); } 1.3 \text{ (CS)}$$

The difference between the values of these coefficients is only $\simeq 10\%$. Thus, if the relation (27) is satisfied for NME calculated in, say, RFSV model [25], it will be difficult to exclude the model CS [26]. The values of the effective Majorana mass which can be obtained from the experimental data in the case of these two models are, however, quite different:

$$|m_{ee}|_{\text{RFSV}} \simeq 2.5 \cdot |m_{ee}|_{\text{CS}}. \quad (31)$$

The observation of $0\nu\beta\beta$ -decay of three (or more) nuclei would be an important tool in a model independent approach to the determination of the value of the effective Majorana mass $|m_{ee}|$ which we have discussed here.

5 Conclusion

The establishment of the nature of the massive neutrinos ν_i would have a profound importance for the understanding of the origin of small neutrino masses and neutrino mixing. The investigation of the neutrinoless double β -decay is the most sensitive method which could allow us to reveal the Majorana nature of the massive neutrinos. Today's limit on the effective Majorana mass, which can be inferred from the study of this process, is $|m_{ee}| \leq (0.3 - 1.2)$ eV. In several experiments now in preparation the sensitivity $|m_{ee}| \simeq$ a few 10^{-2} eV is planned to be reached. If $|m_{ee}|$ is measured, the pattern of the neutrino mass spectrum and, possibly, Majorana CP phase could be revealed. For that not only $0\nu\beta\beta$ -decay must be observed but also nuclear matrix elements must be known. Observation of $0\nu\beta\beta$ -decay of several nuclei could provide a model independent method of testing of NME calculations.

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